Worksheet 6

Monday, October 28, 2019

Math 205

1. You have been assigned into groups of three for the week. Write down your colleagues' names and email addresses.

2. (a) In the xy-plane, sketch the lines $y = 1$, $y = x$, $y = 2x$, and $y = 3x$.
   (b) If we homogenize the first equation, we get $y = z$. The other three equations are already homogeneous. Set $x = 1$ and sketch the four lines in the yz-plane.
   (c) Set $y = 1$ and sketch the four lines in the xz-plane.
   (d) Which line segments correspond to one another in your three drawings?

3. (a) In the xy-plane, sketch the lines $x = 1$ and $x = 2$, and the parabola $y = x^2$.

   (b) Homogenize all three equations. Hint: The third one becomes $yz = x^2$.

   (c) Set $y = 1$ and sketch all three in the xz-plane.

   (d) Set $x = 1$ and sketch all three in the yz-plane.

   (e) Which line segments and curve segments correspond to one another in your three drawings?

   (f) Optional: What if you add the line $x = 0$?

4. (a) In the xy-plane, sketch the circle $(x-1)^2 + y^2 = 1$, which you could rewrite as $x^2 - x + y^2 = 0$.

   (b) Homogenize to get $x^2 - xz + y^2 = 0$. Set $x = 1$ and sketch the curve in the yz-plane. What shape is it?

   (c) Set $y = 1$ and sketch the curve in the xz-plane. What shape is it?

   (d) Which curve segements correspond to one another in your three drawings?

5. If you have time! Or you could revisit this for the final portfolio, if you're interested.

   (a) In 3 dimensions, sketch the hyperboloid of one sheet $z^2 = x^2 + y^2 + 1$.

   (b) Homogenize to get $z^2 = x^2 + y^2 + w^2$. Find another patch in which this looks like a sphere, and the "plane at infinity" $w = 0$ becomes the equator.

   (c) Play a similar game with the elliptic paraboloid $z = x^2 + y^2 + 1$. In one patch, it should look like a sphere tangent to the plane $w = 0$.

   (d) Look at the hyperbolic paraboloid $z = x^2 - y^2$ in several patches.

   (e) Look at the cone $z^2 = x^2 + y^2$ in several patches.