CORRIGENDA: "ON EXTENSIONS OF SIMPLE MODULES OVER SYMMETRIC AND ALGEBRAIC GROUPS"

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We correct some errors and an inconsistency appearing in [1].

1. We are grateful to Kai Meng Tan for pointing out that remark (ii) on page 706 does not make sense, as the partition $\lambda = (1^p)$ is not p-regular. In fact, the remark should be applied to Theorem 2.9, not 2.10.

2. We are grateful to Karin Erdmann and Anne Henke for pointing out that calculations of cohomology in Theorem 3.5 and Corollary 3.6 contain errors. Below is a corrected version of these results. We note that Erdmann and Henke have recently obtained the same results for symmetric and general linear groups using different methods [2].

Theorem 3.5 (i) *Assume* $G = GL_n(F)$, $\lambda = (2^u, 1^{v-u})$, $\mu = (2^r, 1^{s-r})$, $0 \le u \le v \le n$, $0 \le r \le s \le n$, $u \ge r$, *and let* $v - u + 1 = \sum_{i \ge 0} a_i p^i$ *be the* p*-adic expansion. Then* $\mathrm{Ext}^1_G(L(\lambda), L(\mu)) \cong \mathrm{Ext}^1_G(L(\mu), L(\lambda)) = 0$, *unless* $s - v = u - r = (p - a_i)p^i$ *for some* i *such that* $a_i > 0$ *and either* $a_{i+1} < p - 1$ *or* $\min(u, n - v) < p^{i+1}$. *In this case the* Ext *is* 1*-dimensional.*

(ii) *Assume* $G = SL_n(F)$, $\lambda = \omega_u + \omega_v$, $\mu = \omega_r + \omega_s$, $0 \le u \le v \le n$, $0 \le r \le s \le n$, $u \ge r$, *(*ω_0, ω_n *are interpreted as* 0*), and let* $v - u + 1 = \sum_{i \ge 0} a_i p^i$ *be the* p*-adic expansion. Then* $\mathrm{Ext}^1_G(L(\lambda), L(\mu)) \cong \mathrm{Ext}^1_G(L(\mu), L(\lambda)) = 0$, *unless* $s - v = u - r = (p - a_i)p^i$ *for some* i *such that* $a_i > 0$ *and either* $a_{i+1} < p - 1$ *or* $\min(u, n - v) < p^{i+1}$. *In this case the* Ext *is* 1*-dimensional.*

(iii) *Assume* $G = Sp_{2n}(F)$, $\lambda = \omega_u$, $\mu = \omega_r$, $0 \le r \le u \le n$, *(*ω_0 *is interpreted as* 0*) and let* $n - u + 1 = \sum_{i \ge 0} a_i p^i$ *be the* p*-adic expansion. Then* $\mathrm{Ext}^1_G(L(\lambda), L(\mu)) \cong \mathrm{Ext}^1_G(L(\mu), L(\lambda)) = 0$, *unless* $u - r = 2(p - a_i)p^i$ *for some* i *such that* $a_i > 0$ *and either* $a_{i+1} < p - 1$ *or* $u < 2p^{i+1}$. *In this case the* Ext *is* 1*-dimensional.*

(iv) *Assume* $p > 2$, $G = \Sigma_n$, *and* $\lambda = (v, u)$, $\mu = (s, r)$ *be partitions of* n *with* $0 \le u \le v$, $0 \le r \le s$, *and* $u \ge r$. *Let* $v - u + 1 = \sum_{i \ge 0} a_i p^i$ *be the* p*-adic expansion. Then* $\mathrm{Ext}^1_G(D^\lambda, D^\mu) \cong \mathrm{Ext}^1_G(D^\mu, D^\lambda) = 0$, *unless* $u - r = (p - a_i)p^i$ *for some* i *such that* $a_i > 0$ *and either* $a_{i+1} < p - 1$ *or* $u < p^{i+1}$. *In this case the* Ext *is* 1*-dimensional.*

Proof. (i) By Proposition 3.1 and Theorem 3.3, $\mathrm{Ext}^1_G(L(\lambda), L(\mu)) = 0$ unless $\mu = \nu_I(\lambda)$ for a minimal non-empty I in A_λ, in which case the Ext is 1-dimensional. To describe the minimal non-empty sets I in A_λ, recall that any $I \in A_\lambda$ is a union of the form $[i_1, i_2) \cup [i_3, i_4) \cup \cdots$, where $[i_1, i_2)$ clearly belongs to A_λ. So we may assume that $I = [i, j)$, with $a_i \ne 0$ and $a_j \ne p - 1$. Now, if $j = i + 1$, then I is clearly minimal. Let $j > i + 1$. If there exists $k \in (i,j)$ with $a_k < p-1$, then $\delta_{[i,k)} \leq \delta_I$, whence $[i,k) \in A_\lambda$, and I is not minimal. So we may assume that $a_k = p-1$ for all $k \in (i,j)$. In this case $\delta_{[i+1,j)} = p^{i+1}$, and it follows that I is minimal if and only if $p^{i+1} > \min(u, n-v)$. This proves (i).

The proofs of (ii) and (iii) are similar, as is the proof of (iv), except that in (iv) we use Proposition 3.2 in place of Proposition 3.1 and Corollary 3.4 in place of Theorem 3.3. □

Corollary 3.6. (i) *Let $G = SL_n(F)$, $\lambda = \omega_u + \omega_v$ for $0 \leq u \leq v \leq n$, and let $v-u+1 = \sum_{i\geq 0} a_i p^i$ be the p-adic expansion. Then $H^1(G, L(\lambda)) = 0$, unless $n-v = u = (p-a_i)p^i$ for some i such that $a_i > 0$ and either $a_{i+1} < p-1$ or $u < p^{i+1}$. In this case $H^1(G, L(\lambda)) \cong F$.*

(ii) *Let $G = Sp_{2n}(F)$, $\lambda = \omega_u$ for $0 \leq u \leq n$, and let $n-u+1 = \sum_{i\geq 0} a_i p^i$ be the p-adic expansion. Then $H^1(G, L(\lambda)) = 0$, unless $u = 2(p-a_i)p^i$ for some i such that $a_i > 0$ and either $a_{i+1} < p-1$ or $u < 2p^{i+1}$. In this case $H^1(G, L(\lambda)) \cong F$.*

(iii) *Let $p > 2$, $G = \Sigma_n$, $\lambda = (v,u)$ be a partition of n, and $v-u+1 = \sum_{i\geq 0} a_i p^i$ be the p-adic expansion. Then $H^1(G, D^\lambda) = 0$, unless $u = (p-a_i)p^i$ for some i such that $a_i > 0$ and either $a_{i+1} < p-1$ or $u < p^{i+1}$. In this case $H^1(G, D^\lambda) \cong F$.*

References

1. A.S. Kleshchev and J. Sheth, On Extensions of simple modules over symmetric and algebraic groups, *J. Algebra* 221 (1999), 705–722.
2. K. Erdmann and A. Henke, *preprint*, Oxford University, 2000.

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